Nematic order in the multiple-spin exchange model on the triangular lattice

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(Received)

We discuss the possibility of finding n-type nematic order in the vicinity of a FM phase in the multiple–spin exchange model on the triangular lattice. We study this problem both from $S = \infty$ (classical) and $S = 1/2$ (quantum) limits.

The possibility of realizing a quantum spin liquid in a frustrated magnet with antiferromagnetic interactions has been widely discussed. Here we consider the nature of a quantum spin liquid appearing in the vicinity of a *ferromagnetic* phase. Our motivation is the gapless spin liquid observed in two-dimensional (2D) solid ^{3}He on graphite,[1),2)] very close to a ferromagnetic phase. This system is believed to be well described by a multiple–spin exchange (MSE) model on the triangular lattice:

$$\mathcal{H} = J \sum_{\text{N.N.}} P_2 + K \sum_p (P_4 + P_4^{-1}) - H \sum_i S_i^z, \tag{1}$$

where P_n denotes the cyclic permutation operator of n spins. The first summation runs over all nearest neighbor pairs and the second one all four-site minimal diamond plaquettes. In 2D solid ^{3}He, the effective two–spin exchange coupling $J < 0$ is *ferromagnetic* (FM) while the four-spin cyclic exchange $K > 0$ is antiferromagnetic (AF). An exact–diagonalization study of finite–size systems[3)] concluded that the FM phase for small K evolves into a state with zero total spin for large values of K and that, if K is quite large, the ground state is spin liquid with no long–range magnetic order and a spin gap of order $|J|$. How such a spin liquid might form out of a FM state, and what the nature of the spin liquid should be in the vicinity of the FM phase, remain open problems.

Here, we present evidence that the first instability of the FM state with increasing K is against a gapless spin liquid state with nematic order. We consider the n-type spin nematic order parameter, defined by

$$\mathcal{O}^{\alpha\beta}(\boldsymbol{r}_i, \boldsymbol{r}_j) = \frac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \frac{1}{3}\delta^{\alpha\beta}\langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle. \tag{2}$$

This type of spin nematic order has been seen in the $S = 1$ bilinear–biquadratic model.[4)] However, to the best of our knowledge, it has never before been found in a spin $S = 1/2$ frustrated system.

Let us consider first the classical limit of the MSE model on the triangular lattice. A mean–field argument and its extension[5)] showed that a highly degenerate

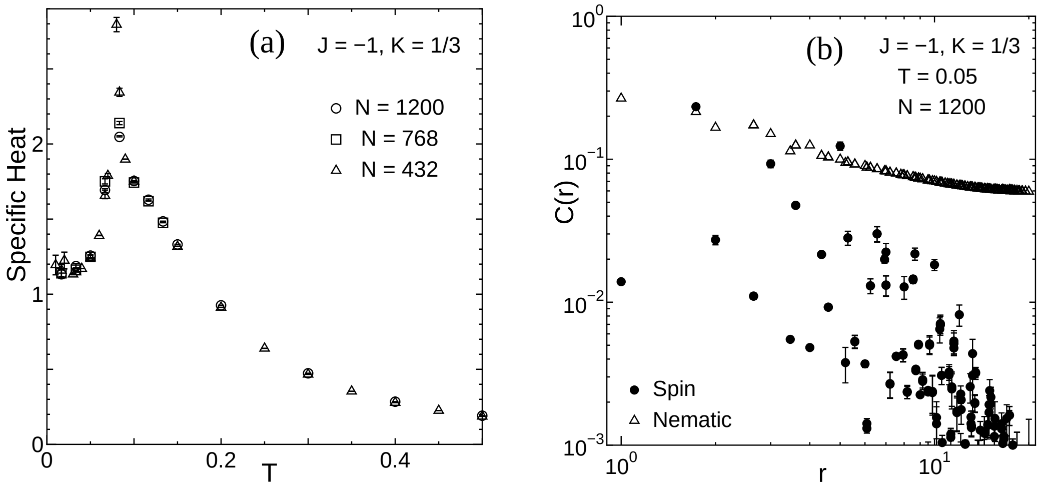

Fig. 1. (a) Temperature dependence of specific heat in the classical MSE model for $J = -1$, $K = 1/3$, $H = 0$ with peak corresponding to the onset of nematic correlations. Results are shown for systems of $N = 432$, 768 and 1200 spins. (b) Decay of spin (circle) and nematic (triangle) correlations at $T = 0.05$ for the classical MSE model with $J = -1$, $K = 1/3$, $H = 0$, and $N = 1200$. All temperatures are measured in units of $|J|$.

phase appears in the wide parameter range $-1 < K/J < -1/4$. This set includes all parameters relevant to 2D solid ^{3}He in the low density limit, and extends up to the classical boundary of the FM phase at $K/J = -1/4$. Among the many possible classical ground states, collinear states are most likely to be favored by quantum or thermal fluctuations and the two most characteristic collinear ground states are the UUUD and UDDD states with exactly half the saturation magnetization.[5)] However, because it costs no energy to add a straight domain wall between these states, at any finite temperature the system will gain entropy by breaking up into stripe–like UUUD and UDDD domains. For this reason we do not expect any long range spin–spin correlations to form in this system, even in the limit $T \to 0$.

However we *do* expect that the spins will remain collinear, and that this will lead to long ranged spin nematic correlations of the form $\mathcal{O}^{\alpha\beta}(\boldsymbol{r}_i, \boldsymbol{r}_j)$, above. To test this argument, we performed MC simulations of the classical model and found that nematic correlations are enhanced at low temperatures, while spin correlations decay quite rapidly. (See Fig. 1.) The onset of nematic correlations is signaled by a peak in the specific heat whose height remains finite as the system size is increased. Of course, true long ranged nematic order cannot be achieved in 2D at any finite temperature because the director (here, the axis of collinearity) breaks a continuous symmetry.

We now turn to the experimentally relevant quantum limit, i.e. the $S = 1/2$ MSE model, and consider the nature of the first instability of the FM as K is increased. In the absence of magnetic field, the entire one magnon spectrum

$$\epsilon(\boldsymbol{k}) = -2(J + 4K)\{3 - \cos(\boldsymbol{k}\cdot\boldsymbol{e}_1) - \cos(\boldsymbol{k}\cdot\boldsymbol{e}_2) - \cos[\boldsymbol{k}\cdot(\boldsymbol{e}_1 - \boldsymbol{e}_2)]\} \qquad (3)$$

vanishes at the classical phase boundary $K/J = -1/4$, reflecting the extensive de-

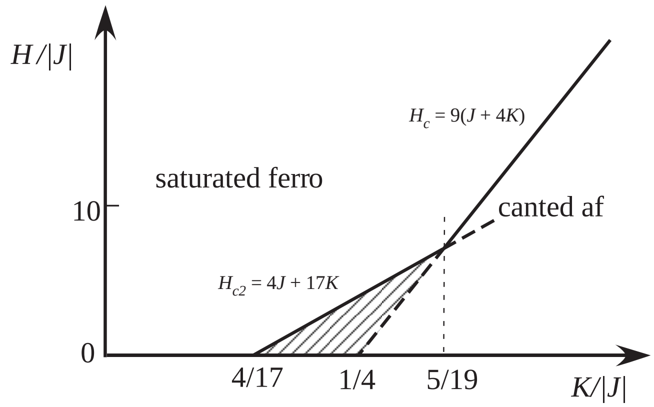

Fig. 2. Saturation fields: the parameter $K/|J|$ vs. the one– and two–magnon saturation fields H_c and H_{c2}. In the shaded region, the bi-magnon bound states have a negative energy relative to the saturated FM state, while individual magnons have a positive energy with a finite positive stiffness. Thus the partially polarized spin state in the shade should show BEC of bound bi–magnons.

generacy of the classical disordered phase beyond it. For large values of applied magnetic field, this degeneracy is lifted and the first instability of the saturated FM state is against a three–sublattice canted AF state. However for smaller applied fields, the system can gain kinetic energy by allowing pairs of flipped spins to move together through repeated use of the cyclic exchange process. This prompts us to consider the following trial bi–magnon bound state:

$$a(\boldsymbol{k})|1(\boldsymbol{k})\rangle + b(\boldsymbol{k})|2(\boldsymbol{k})\rangle \tag{4}$$

with

$$|1(\boldsymbol{k})\rangle = N^{-1/2} \sum_i S_i^- S_{i+e_1}^- e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_i + \boldsymbol{e}_1/2)} |PF\rangle,$$
$$|2(\boldsymbol{k})\rangle = N^{-1/2} \sum_i S_i^- S_{i+e_2}^- e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_i + \boldsymbol{e}_2/2)} |PF\rangle.$$

Despite the fact that the static interaction between pairs of magnons is repulsive, for $K/J < -4/17$, there exist trial solutions with a negative energy relative to the saturated FM state, i.e. bi–magnon bound states below the FM continuum. Thus the first instability of the saturated FM state in zero field is against the spontaneous creation of bound bi–magnon pairs. Extending this calculation to finite magnetic field, we obtain the phase diagram shown in Fig. 2.

In order to better understand the unsaturated phase bordering on the FM phase let us consider the dynamics of bound bi-magnons. We know that pairs of flipped spins have repulsive interactions and a definite (positive) hopping amplitude. From this we can write down an effective Hamiltonian for bound bi–magnons, which has a form similar to an AF XXZ model. We therefore anticipate that the density of bound bi-magnons increases continuously upon decreasing the magnetic field from

the critical value H_{c2}, and that there is a second order quantum phase transition from the saturated to the unsaturated state.

To be certain that bound bi–magnons are the elementary particles of the unsaturated state, we need to rule out the existence of larger magnon bound states. Cooperative motion of more than three flipped spins can occur in high order perturbation theory in K. However the repulsive interaction between magnons is first order in K, so we do not, in general, expect larger bound states to be stable. From these arguments we conclude that the unsaturated states close to the saturation field are characterized by the Bose–Einstein condensation (BEC) of a low density of bound bi-magnons.

Finally we consider the nature of this bound bi–magnon condensate. The magnon pairing operator is given by

$$\mathcal{O}_{\text{pair}}(\boldsymbol{r}_i, \boldsymbol{r}_j) = s_i^- s_j^- . \tag{5}$$

This is related to the nematic operators through

$$\mathcal{O}_{\text{pair}} = s_i^- s_j^- = \mathcal{O}_N^{xx} - \mathcal{O}_N^{yy} - 2i\mathcal{O}_N^{xy}, \tag{6}$$

where we have omitted site indices. The real part of the pairing operators corresponds to the nematic operator $\mathcal{O}_N^{xx} - \mathcal{O}_N^{yy}$ and the imaginary part to $-2\mathcal{O}_N^{xy}$. These two components give nematic order parameters for the systems under the magnetic field. Thus, the BEC of bound bi–magnons is equivalent to the emergence of long–ranged spin nematic order.

Our conclusion is that the MSE model on the triangular lattice in the vicinity of the saturated FM phase shows long ranged spin nematic order in one of two channels

$$\begin{aligned} \lim_{|r|\to\infty} \langle [\mathcal{O}^{xx}(\boldsymbol{0}, \boldsymbol{e}_1) - \mathcal{O}^{yy}(\boldsymbol{0}, \boldsymbol{e}_1)][\mathcal{O}^{xx}(\boldsymbol{r}_i, \boldsymbol{r}_i + \boldsymbol{e}_1) - \mathcal{O}^{yy}(\boldsymbol{r}_i, \boldsymbol{r}_i + \boldsymbol{e}_1)]\rangle \\ \sim c_1(1 - m/m_{sat}) \end{aligned} \tag{7}$$

or

$$\lim_{|r|\to\infty} 4\langle \mathcal{O}^{xy}(\boldsymbol{0}, \boldsymbol{e}_1)\mathcal{O}^{xy}(\boldsymbol{r}_i, \boldsymbol{r}_i + \boldsymbol{e}_1)\rangle \sim c_1(1 - m/m_{sat}), \tag{8}$$

where m (m_{sat}) denotes the (saturated) magnetization and c_1 a finite constant. In these states, spin correlations decay exponentially even at zero temperature, making them as true spin–liquids. The rate at which the spin correlations decay is set by the binding energy of bound bi–magnon pairs, which vanishes at the boundary with the saturated FM phase.

Correlated hopping processes[7)] lead to a similar BEC of bound bi–magnons in the low-magnetization regime of the 2D Shastry–Sutherland model[8)] . This can also be regarded as a spin nematic state. However, nematic order may be difficult to observe in the "Shastry–Sutherland" compound $SrCu_2(BO_3)_2$ because of the presence of Dzyaloshinsky–Moriya interactions which break the bi–magnon pairs. The spin interactions in solid ^{3}He, on the other hand, are purely isotropic in nature. We therefore expect that nematic order *is* experimentally accessible for a range of densities of 2D solid ^{3}He in applied magnetic field.

We will return to this issue, and present further analytic and numerical results for nematic order neighboring ferromagnetism in the MSE model, elsewhere.[6)]

Acknowledgements

N.S. would like to thank Michel Roger for many helpful discussions about ^{3}He and multiple spin exchange.

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